

On Black Hole Remnants

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Abstract

We introduce two models for a planck scale black hole remnant (Planckon), which can hold arbitrarily large information, while keeping a vanishing coupling and discuss their physical properties.

May, 2007

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1 Introduction

1.1 The Black-Hole Information Paradox

Hawking's semi-classical calculation of black holes radiance [1] led to the possibility of black hole evaporation. This in turn, led to the conclusion that information may be

lost forever, in practice, from the world - this is the black-hole information paradox.

The paradox may be described in the following manner:

In 1975 Hawking calculated the emission of radiation from a stationary classical black hole. The calculation was done using a semi-classical calculation for non-interacting matter fields propagating over classical Schwarzschild black hole geometry.

This calculation resulted in a surprising discovery that a black hole not only radiates, but radiates as a black-body with temperature of:

$$T_{BH} = \frac{\hbar c^3 \kappa}{2\pi k_B G} \quad (1)$$

Where κ is the surface gravity (for a Schwarzschild black hole $\kappa = \frac{1}{4M}$).

Since the emitted radiation is a black-body radiation it is exactly thermal (uncorrelated), in particular, the emitted radiation does not depend on the structure of the collapsed body that formed the radiating black-hole (the collapsed body is trapped behind the horizon and is unable to influence anything outside the horizon).

The radiation depends only on the geometry of the black hole outside the horizon (depends only of the mass, angular momentum, charge etc. of the black hole) and can not depend or be correlated with the collapsed body (there might be some weak correlations since Hawking's calculation is not exact).

By itself, the fact that the radiation outside the black hole is thermal is not too disturbing, since it is only a part of a whole quantum system. Part of the quantum system is inaccessible, as it is trapped behind the horizon. There are some correlations between the degrees of freedom, which are accessible outside the horizon and the ones inside the horizon. Because of the correlations, an observer outside the horizon, detecting the quantum fields (degrees of freedom which were radiated), will not be able to determine the exact initial quantum state of the collapsed body and will only detect a mixed state.

During the radiation process the black hole radiates its energy (mass) away and so, if one waits long enough, the black hole will evaporate completely, leaving behind only the thermal radiation. The thermal radiation, which is a mixed state, is now the whole system. The consequences for such a process are that beginning with a pure state does not allow one to predict with certainty, what will the final quantum state be (the final system is a mixed state so one can only assign probabilities to different final states).

The evaporation of a black hole, as described above, results in a paradox. According to the laws of quantum mechanics and field theory - if one completely specifies the initial state of a system and knows all the stages of its evolution, one knows the final state of the system at all future times (this is the unitarity postulate of quantum mechanics, which states that a system in a pure state will stay in a pure state).

This paradox is known as "The Information Loss Paradox", since from an initially pure state, which has zero entropy, one ends up with a mixed state, which has non-vanishing entropy. Such a process where information is lost indicates a non-unitary evolution, which contradicts the laws of quantum physics.

Hawking showed, that the emitted thermal (or nearly thermal) radiation from the evaporating black hole, carries a huge amount of entropy that can be estimated by:

$$S \sim \frac{M_0^2}{M_{pl}^2} \quad (2)$$

Such an evolution of a black hole, from a pure state into a mixed state, results in a fundamental loss of information:

$$\Delta I = -\Delta S \sim -\frac{M_0^2}{M_{pl}^2} \quad (3)$$

The source of this missing information is the correlation between particles coming out of the black hole and particles falling into the black hole.

The semi-classical calculation is valid until the black hole reaches the Planck scale, where quantum gravity effects that break the semi-classical approximation, may affect the process. The Planck scale, which is given by Planck's mass, time and length, can be formed by combining the gravitational constant G , the quantum of action \hbar and the speed of light c in a unique way.

The Planck units are:

$$\begin{aligned} M_{pl} &= \sqrt{\frac{\hbar c}{G}} \approx 1.22 \cdot 10^{19} GeV \approx 2.17 \cdot 10^{-5} [gr] \\ t_{pl} &= \sqrt{\frac{\hbar G}{c^5}} \approx 5.4 \cdot 10^{-44} [sec] \\ l_{pl} &= \sqrt{\frac{\hbar G}{c^3}} \approx 1.62 \cdot 10^{-33} [cm] \end{aligned} \quad (4)$$

Since quantum gravity is expected to play a key role in the evaporation process, one

may hope, that a resolution of the black hole information paradox may be found as the black hole (mass and length) approaches the planck scale by some , yet unknown, quantum gravitational effect.

1.2 Possible Solutions To The "Black Hole Information Paradox"

Hawking's semi-classical calculation indicates the breakdown of predictability and unitarity in physics. Yet the calculation is valid only at length scales larger than the Planck scale. One might hope that the solution to the apparent paradox will appear at the Planck scale and thus will produce some clues, as to how to construct a quantum gravity theory or at least shed light on some of its features.

There are three main approaches to find a solution to the black hole information paradox:

The first accepts the information loss. The second asserts that the information is retrieved during the evaporation process or via effects, which occur around the Planck scale. The last solution relies on the possible existence of Planck scale remnants.

The next few paragraphs will contain a brief discussion on the first two approaches and the rest will focus on the last solution, which is the basis of this article.

Information loss:

This solution tries to implement information loss into physics and especially into quantum physics. The primary attempts are to change or generalize the unitarity postulate of quantum mechanics to allow non-unitary evolution. An example for such an extension of quantum mechanics was offered by Hawking [5], who suggested replacing the usual S matrix of quantum mechanics (which maps a pure state only to another pure state) with a super-scattering matrix S , which acts on density matrix (instead of on state vectors) in the following way:

$$\rho_{ab}^{final} = S_{ab}^{cd} \rho_{cd}^{initial} \quad (5)$$

The super-scattering operator S acts on a density matrix and maps it into another density matrix in a non-unitary way and thus can increase the entropy. In particular, the operator S can act on a pure state and map it into a density matrix (mixed state).

The main drawbacks of accepting information loss as an unavoidable feature of quantum gravity are that no one has yet found a way to incorporate non-unitarity into consistent physical theories that gives satisfactory results. Furthermore, the fact that transmitting information requires energy indicates that losing information might be connected to violation of energy and momentum conservation.

Let us assume that a black hole forms and then evaporates in a time Δt - then from the uncertainty principle one has

$$\Delta E \geq \frac{1}{\Delta t} \quad (6)$$

This shows that the minimum of energy loss in the process should be of order $\frac{1}{\Delta t}$.

In quantum theory such formation and evaporation should occur all the time as virtual processes. The amplitude for such processes approaches unity when the size of the loop approaches the Planck scale (there is no smaller dimensionless number to suppress it). Thus, one would expect Planck size energy violations with characteristic time of the order of Planck time. This would cause the world to seem as a thermal bath of Planck temperature, which is obviously not the case.

Information retrieval:

This line of thought suggests that unitarity is not violated, usually by suggesting that the information about the state of the collapsing matter is encoded in the emitted Hawking radiation. Alternatively, by suggesting that the information comes out in a final burst when the black hole reaches the Planck scale (Planck mass).

Both suggestions have their drawbacks:

The first suggestion implies that matter behind the horizon affects matter outside the horizon, despite the fact that the two regions of space-time are space-like separated. Hence, one has to give up the notion of locality and causality or at least change them radically (this idea also contradicts Hawking's calculation [1]).

The second suggestion raises problems with energy conservation since one can show that the information does not have enough energy to exit in a final burst:

The energy of the black hole in the Planck scale is $E_{pl} \sim M_{pl}$ and the information to be transmitted is of the order of $\frac{M_{pl}^2}{M_{pl}^2}$. Because of the uncertainty principle, the only way to transmit a lot of information with little energy is to transmit the information over a long period of time Δt .

An estimate of the time required for the transmission given Planck energy is:

$$t \sim \left(\frac{M_0}{M_{pl}} \right)^4 t_{pl} \quad (7)$$

This time exceeds the age of the universe for most black holes and thus, one is drawn to the possibility of stable or long-lived remnants of Planck mass.

1.3 The Remnant Solution And Its Drawbacks

Another possibility is to assume that when the black hole reaches the Planck scale, it tunnels due to quantum effects into a stable or nearly-stable particle which keeps the information about the initial state.

As was implied above - the information can not come out at the end of the evaporation with a final burst:

For a black hole with mass M , the emitted radiation state must contain energy M inside a sphere, whose radius is comparable to the hawking evaporation time of the black hole $t_{Hawking} \sim M^3$ (Working in the natural units in which the Plank mass $M_{pl} = 1$).

The remnant can decay into $N \simeq \left(\frac{M_0}{M_{pl}} \right)^2$ quanta [7], but such a decay is highly suppressed because of the tiny wave function overlap factor. The reason for this small overlap is, that the only way to transmit all the information with small available energy, is to use very low energy (corresponding to very long wavelength) states. The overlap between the states wave-function and the remnant wave-function is very small.

To quantify the argument above let us examine the average wavelength of the emitted quanta [7]:

The average wavelength of the final N emitted quanta is

$$\lambda \approx \left(\frac{M_{pl}}{N} \right)^{-1} \approx N R_{pl} \quad (8)$$

one can easily see that the wavelength of the emitted wave is larger by a factor of N than the size of the decaying system (Planck size black hole has a radius of $\sim R_{pl}$).

The "wave-function overlap" between each of the emitted quanta and the decaying system is therefore $f = \frac{R_{pl}^3}{\lambda^3} \approx N^{-3}$. The simultaneous emission of N quanta is suppressed

by a tiny factor of $f^N \approx N^{-3N}$. This tiny factor will render a planck scale remnant practically stable.

The remnant idea comes as another way to retain the unitarity postulate of quantum mechanics and avoid the final burst of information by leaving behind a long lived remnant.

Considering the suppression factor above, estimates of the remnant evaporation time $t_{remnant}$ [2, 17] yield a lower bound for $t_{remnant}$:

$$t_{remnant} \geq \left(\frac{M_0}{M_{pl}} \right)^4 t_{pl} \quad (9)$$

In the last formula the Planck factors were reinstated.

The long evaporation time can be understood as the decay time required for a very long wavelength mode. One should notice that $t_{remnant} > \text{"age of the universe"}$ which validates the claim that the planckon is virtually stable.

One can now see that in order to retain the unitarity postulate one has to assume a stable black hole remnant which should have mass that is equal or near the Planck mass.

Black hole Planck scale remnants were given the name Planckons [7], a name that will be used from now on.

One can also ask whether a Planckon can be charged (either electric, weak, color, etc.) or have angular momentum, since the evaporating black hole can be spinning and/or charged (Kerr black hole). The Hawking radiation of a Kerr black hole is such that the black hole emits its angular momentum and charge by radiating its charge and angular momentum away, creating charged or spinning particles. Thus, when the black hole reaches the Planck scale, one is left with a Schwarzschild black hole ¹ [18]. Even if one does end up with a charged or spinning Planckon, the Planckon will lose its charge or angular momentum by pair creation of particles and "swallowing" particles with opposite sign and angular momentum. Therefore a Planckon should have no charge or angular momentum.

Since a black hole can be arbitrarily large - to be able to store (encode) the informa-

¹ Since the rate of such a process for charged black hole is proportional to some power of α the time for a black hole to loss its charge (this is true for any quantum charge) is much smaller than the time for the black hole to reach the Planck scale

tion about the original state of a black hole with a Planckon, infinite different species of Planckons must exist (Planckons have infinite degeneracy). Such a large reservoir of quantum states implies huge entropy, which is much larger than the usual black hole entropy $S_{bh} = \frac{A}{4}$. The excess entropy may be expressed by the integration constant usually omitted in the derivation of the black-hole entropy from $dS = \frac{dE}{T}$. With the presence of an integration constant C the black hole entropy becomes:

$$S_{bh} = \frac{A}{4} + C \quad (10)$$

If one considers an infinite (or very large) C the last stage of the black hole evaporation should be modified by allowing the black hole to tunnel into a Planckon (i.e a remnant with infinite degeneracy).

The main argument raised against the Planckon paradigm is that having an infinite number of Planckons with, approximately, the same mass will lead to a divergence in any process with energy higher than M_{pl} . Since there must be a tiny, non zero, amplitude of Planckon production and since one must sum over all possible (infinite) species of Planckons, one ends up with an infinite production rate, which will cause the universe to be unstable to instantaneous decay into remnants (which is evidently not the case).

Such an infinite production rate will also plague the coupling of Planckons to soft quanta ($wavelength \gg l_{pl}$), where Planckons can be described by an effective theory, in which the Planckons are described by a point-like object. Thus the coupling to soft gravitons, for example, will depend only on its mass and not on its internal structure or information content and again the infinite number of species will cause the luminosity to be infinite and a divergence in the graviton propagator will occur. Such divergences should have great impact on low energy physics (due to the coupling between soft quanta and Planckons).

Another argument raised against the Planckon is that a planck scale remnant cannot hold the required information because of entropy bounds relations between entropy and energy [12].

A physical model for a Planckon should deal with the above problems without invoking any new and unfamiliar physics. A discussion on the way the model deals with

the above problems can be found in 5.

For further discussion see the excellent reviews [2, 3, 4, 5, 6, 7] and references therein.

2 Models For The Planckon

This section contains a development of a consistent physical models for a Planckon using semi-classical methods (i.e. Quantum fields on curved space-time, WKB approximation etc.).

Such a model will include the main features of the Planckon as mentioned in 1.3 (neutral particle with infinite degeneracy) and will also provide an effective description as to how the Planckon avoids the estimate of infinite production rates, despite its inherent infinite degeneracy.

The model is based on [7], where the basic properties of the Planckon were outlined and on [8, 9], where a precursor model (that of the "Achronon") is outlined and the possibility of the existence of the Planckon is briefly discussed.

2.1 General Properties Of The Models

As the evaporating black hole approaches the Planck scale and the Compton wavelength of the remaining black hole exceeds its Schwarzschild radius, quantum effects (especially the uncertainty principle) become important and a quantum treatment of the system is required.

From now on, unless stated otherwise, the natural units will be used $c = G = \hbar = 1$ and the signature is of the form $(+, -, -, -)$.

In the models suggested the black hole (whose mass is approximately m_{pl}) tunnels into a specific state, in which the mass is distributed at a distance $\Delta \ll 1$ from its Schwarzschild horizon.

The proper physical description of the Planckon is a soliton with mass of the order of m_{pl} , but, since the knowledge of the quantum treatment of a soliton is limited to a perturbative treatment, i.e., expanding the corrections in orders of $\frac{1}{M_{soliton}}$. At the planck scale the perturbative expansion breaks down, since $\frac{1}{M_{soliton}} \sim \frac{1}{m_{pl}} \sim 1$ and a different treatment is required. In the models suggested, the soliton problem

was avoided by describing the mass configuration of the Planckon with a spherical-symmetric scalar field. The scalar field generates the classical geometry (metric). The quantum corrections to the planckon are given by quantum fields propagating over the classical geometry. For simplicity only massless quantum scalar and fermion fields ² are considered. The mass configurations are chosen specifically to produce a tiny (almost zero) g_{00} . Looking at the field equations one can easily see that the time dependence of a field is proportional to some power of g_{00} , which means that the fields are almost static (this time independence is a manifestation of the gravitational time dilation). This g_{00} time dependence also appears in the Einstein field equations, i.e., the metric is almost static. Since the time dependence of each field is proportional to g_{00} each vertex will carry a power of g_{00} and quantum corrections to the soliton will take the form of a perturbative expansion in g_{00} .

The classical scalar field satisfies the following conditions:

$$\partial_t g_{\mu\nu} = 0 ; -g^{rr} = 1 - \frac{2M(r)}{r} ; \partial_t \phi = 0 ; \partial_\varphi \phi = 0 ; \partial_\theta \phi = 0 \quad (11)$$

These conditions ensure that the metric and the scalar fields are spherically symmetric and are "frozen" in time as described above. The uncertainty principle together with the g_{00} time dependence will prevent the mass configuration from collapsing due to gravitational force.

Note that this type of configuration cannot be produced by collapse from infinity and can be reached only by tunneling [8, 9].

The expression for the energy-momentum tensor of a scalar field is:

$$T_\nu^\mu = g^{\mu\lambda} \partial_\lambda \phi \partial_\nu \phi - \frac{1}{2} g^{\mu\lambda} g_{\nu\lambda} g^{\rho\sigma} \partial_\rho \phi \partial_\sigma \phi \quad (12)$$

The above particular conditions generate an energy-momentum tensor with the following property:

$$T_r^r = -T_t^t \quad (13)$$

The T_t^t term will be given by the mass density distribution of the specific model and

²The need for super-symmetry will be explained within the context of the models

the metric will be given by the Schwarzschild solution for the given energy-momentum tensor:

$$-g^{rr} = 1 - \frac{2M(r)}{r} ; g_{tt} = \left(1 - \frac{2M(r)}{r}\right) \exp\left(-8\pi \int_r^\infty r'^2 \frac{2T_t^t}{r' g_{rr}} dr'\right) \quad (14)$$

The effective potentials quantum scalar and fermion fields (massless S-wave fields) experience, when propagating over curved background, are:

$$V_{scalar}(r) = \frac{1}{2r} \partial_r ((-g^{11}) g_{00}) \quad (15)$$

$$V_{fermion} = W^2(r) \pm \frac{\partial W(r)}{\partial \rho} \approx \frac{k^2}{r^2} g_{00} \quad (16)$$

Where in the fermion case $W(r) = \frac{|k|}{r} \sqrt{g_{00}}$ is a super symmetric potential and thus only the $W^2(r) = \frac{k^2}{r^2} g_{00}$ term contributes [10, 11].

It is shown, that for a given quantum field, there is a nearly infinite number of possible quantum excitations.

In this paper the physical properties of two possible models, which allow the Planckon to have infinite degeneracy but finite production rate and are also singularity free, are investigated.

3 Potential Well Model For The Planckon

In this model the planckon has a total mass M which is located at $r = 2M + \Delta$ (a distance Δ from the Schwarzschild horizon $r = 2M$).

The mass distribution is:

$$m(r) = M \cdot \Theta(r - 2M - \Delta) \quad (17)$$

This mass distribution gives the following energy distribution:

$$T_0^0 = \frac{1}{4\pi r^2} \partial_r M(r) = \frac{M}{4\pi r^2} \delta(r - 2M - \Delta) \quad (18)$$

The metric generated by this energy momentum tensor is:

$$g_{00} = \begin{cases} e^{-\frac{4M}{\Delta}} & r \leq 2M + \Delta \\ 1 - \frac{2M}{r} & 2M + \Delta < r \end{cases} ; g_{11} = \begin{cases} 1 & r \leq 2M + \Delta \\ -\left(1 - \frac{2M}{r}\right)^{-1} & 2M + \Delta < r \end{cases} \quad (19)$$

as can be seen the model contains no singularities and no horizons.

The quantum corrections to the Planckon are given by the quantum fields propagating in the volume trapped by the classical mass configuration.

The model is described by first evaluating the energy correction due to a scalar field and than expanding the model to include the super-symmetric fermionic partner, in order to cancel the divergence in the self energy.

3.1 Self Energy Of A Scalar Field

The scalar field equation of motion will be of the form:

$$\partial_\rho^2 h(r) - \omega^2 h(r) + \frac{1}{2r} \partial_r (-g^{11} g_{00}) h(r) = \partial_\rho^2 h(r) - \omega^2 h(r) = 0 \quad (20)$$

Where the following definitions are used:

$$\rho = \sqrt{-g_{11} g^{00}} r = \sqrt{g^{00}} r = r e^{\frac{2M}{\Delta}} \quad (21)$$

The potential is located at:

$$\rho(2M + \Delta) = (2M + \Delta) e^{\frac{2M}{\Delta}}. \quad (22)$$

Assuming the boundary conditions:

$$h(0) = h(\rho(2M + \Delta)) = 0 \quad (23)$$

The energy eigenvalues are the same as for a potential well. The self energy will be of the form:

$$E = \frac{1}{2} \sum \omega = \frac{1}{2} \sum \frac{n\pi}{\rho(2M + \Delta)} = \frac{1}{2} \sum_{n=1}^{\frac{\rho(2M + \Delta)}{\pi} \sqrt{V(2M + \Delta)}} \frac{n\pi}{\rho(2M + \Delta)} \propto \rho(2M + \Delta) \quad (24)$$

The self energy “diverges” as $\rho(2M + \Delta) = (2M + \Delta)e^{\frac{2M}{\Delta}}$.

To make this model finite one has to invoke super-symmetry. We emphasize that exact super-symmetry is not required, but only that there is an equal number of bosonic and fermionic degrees of freedom (need not have the same energy levels). The model is modified only by the minimal modifications needed to make it super-symmetric, which means replacing the scalar field with a complex scalar field (the self energy will grow by a factor of 2) and adding a weyl fermion (the super-symmetric partner of the complex scalar). The total self energy will be the sum of the two contributions of the self energies of the fields.

3.2 Self Energy Of A Fermion In The Potential Well Model

The fermion self energy in a spherical potential well will be derived by following the derivation of the solution for the MIT bag model in [14] and [15, 16].

The massless Dirac equation is:

$$\gamma^\mu \nabla_\mu \psi = 0 \quad (25)$$

where $\nabla_\mu = \partial_\mu - \Gamma_\mu$ and Γ_μ is the spin-connection in the vierbeins method.

The solutions for ψ are of the form:

$$\psi = \begin{pmatrix} g(r)Y_{[l_a \frac{1}{2}]jm_j} \\ \pm f(r)Y_{[l_b \frac{1}{2}]jm_j} \end{pmatrix} \quad (26)$$

the \pm is defined for:

$$k = \mp(j + \frac{1}{2}) = \begin{cases} -(j + \frac{1}{2}) = -(l_a + 1) & j = l + \frac{1}{2} \\ (j + \frac{1}{2}) = l_a & j = l - \frac{1}{2} \end{cases} \quad (27)$$

Where the following definitions have been used:

$$j = (l_a + \frac{1}{2}) ; l_b = (l_a + 1) \text{ for } k < 0 \quad (28)$$

$$j = (l_a - \frac{1}{2}) ; l_b = (l_a - 1) \text{ for } k > 0 \quad (29)$$

k is the dirac quantum number which differentiates the two states of opposite parity for each value of j.

If one defines $G(r) = r \cdot g(r)$ and $F(r) = r \cdot f(r)$ one gets the following equations:

$$\left(-\sqrt{(-g^{11})} g_{00} \partial_r - \sqrt{g_{00}} \frac{k}{r} \right) G + \omega F = 0 \quad (30)$$

$$\left(\sqrt{(-g^{11})} g_{00} \partial_r - \sqrt{g_{00}} \frac{k}{r} \right) F + \omega G = 0 \quad (31)$$

Substituting the last 2 equations into each other one gets:

$$\partial_\rho^2 F + \left(\omega^2 - \frac{k(k-1)}{\rho^2} \right) F = 0 \quad (32)$$

$$\partial_\rho^2 G + \left(\omega^2 - \frac{k(k+1)}{\rho^2} \right) G = 0 \quad (33)$$

Where ρ has the is the same as in (24) (the same as for the scalar field).

Since $j = (l_a \pm \frac{1}{2})$ $k = \mp (j + \frac{1}{2})$ one has:

$$k = -(l_a + 1) \Rightarrow \begin{cases} k(k+1) = l_a(l_a + 1) \\ k(k-1) = (l_a + 1)(l_a + 2) = l_b(l_b + 1) \end{cases} \quad (34)$$

$$k = l_a \Rightarrow \begin{cases} k(k+1) = l_a(l_a + 1) \\ k(k-1) = l_a(l_a - 1) = l_b(l_b + 1) \end{cases} \quad (35)$$

If one defines $\rho = \omega \rho'$, the equations will take the general form of:

$$\left(\frac{\partial}{\partial \rho^2} - \frac{l(l+1)}{\rho^2} + 1 \right) u_l = 0 \quad u_{l_a} = G ; \quad u_{l_b} = F$$

Where u_l is the solution for the one dimensional radial equation - in this case u_l is the spherical Bessel function and since one only considers solutions, which are regular at the origin, one has to choose:

$$\frac{G(\rho)}{\rho} = j_{l_a}(\rho) ; \quad \frac{F(\rho)}{\rho} = \mp j_{l_b}(\rho)$$

Where the sign in $F(\rho)$ are for $-(l_a + 1)$ and the $+$ sign is for $k = l_a$.

Incorporating the results into (26) one obtains:

$$\psi = N \begin{pmatrix} j_{l_a}(\omega\rho)Y_{[l_a\frac{1}{2}]jm_j} \\ -j_{l_b}(\omega\rho)Y_{[l_b\frac{1}{2}]jm_j} \end{pmatrix} \quad (36)$$

Where N is a normalization factor.

Our boundary conditions, to ensure the confinement of the fermion field inside $r = 2M + \Delta$, are:

1. $\iint \bar{\psi} \left(\vec{\gamma} \cdot \frac{\vec{r}}{r} \right) \psi r^2 d\Omega \quad r = 2M + \Delta$
2. $\iint \bar{\psi} \psi r^2 d\Omega = 0 \quad r = 2M + \Delta$

The first condition ensures that there is no probability density current in the radial direction from the sphere of radius $r = 2M + \Delta$, while the second condition ensures that the Lorentz scalar quantity $\bar{\psi}\psi$ (the probability of finding the particle) will be zero over a sphere with radius $r = 2M + \Delta$.

The first condition is satisfied automatically from the orthonormality of the spherical harmonics.

Putting the solutions into the boundary condition and defining $a = (2M + \Delta) e^{\frac{2M}{\Delta}}$ the second condition becomes:

$$\begin{aligned} & \iint a^2 \left((j_{l_a}(\omega a))^2 \left| Y_{[l_a\frac{1}{2}]jm_j} \right|^2 - (j_{l_b}(\omega a))^2 \left| Y_{[l_b\frac{1}{2}]jm_j} \right|^2 \right) d\Omega \\ & = ((j_{l_a}(\omega a))^2 - (j_{l_b}(\omega a))^2) = 0 \end{aligned} \quad (37)$$

equation (37) is satisfied if:

$$j_{l_a}(\omega a) = \frac{k}{|k|} j_{l_b}(\omega a)$$

Let us denote the n^{th} solution of this equation for a specific value of k as $\chi_{n,k} = \omega_{n,k}a$ and so the energies will be of the form:

$$\omega_{n,k} = \frac{\chi_{n,k}}{a}$$

Since only orbital momentum $l=0$ is considered, the interest is limited to the case where $k = -1$ ($j = \frac{1}{2}, l_a = 0, l_b = 1$), which gives the equation:

$$j_0(\chi_{n,-1}) = j_1(\chi_{n,-1})$$

The last equation can be written as follows:

$$\tan(\chi_{n,-1}) = -\frac{\chi_{n,-1}}{\chi_{n,-1} - 1} \quad (38)$$

The first few numerical solutions to (38) are:

$$\chi_{1,-1} = 2.0427869 \quad \chi_{2,-1} = 5.396016118 \quad \chi_{3,-1} = 8.577558785 \quad \chi_{4,-1} = 11.73650396$$

One can easily show that the distances between the solutions converge rapidly (from above) to $n\pi$, so one can approximate the energy levels by (a lower bound):

$$E_f = \sum_{n=1}^{\frac{a}{\pi}\sqrt{V(2M+\Delta)}} \frac{\chi_{n,-1}}{a} \approx \sum_{n=1}^{\frac{a}{\pi}\sqrt{V(2M+\Delta)}} \frac{2.043 + (n-1)\pi}{a} = \sum_{n=1}^{\frac{\rho(2M+\Delta)}{\pi}\sqrt{V(2M+\Delta)}} \frac{2.043 + (n-1)\pi}{\rho(2M+\Delta)}$$

3.3 Total energy of the super-symmetric potential well model

The total self-energy (bounded from above) of the model will be:

$$\begin{aligned} E_{self} &= E_s - E_f < \sum_{n=1}^{\frac{\rho(2M+\Delta)\sqrt{V(2M+\Delta)}}{\pi}} \frac{n\pi}{\rho(2M+\Delta)} - \sum_{n=1}^{\frac{\rho(2M+\Delta)\sqrt{V(2M+\Delta)}}{\pi}} \frac{2.043 + (n-1)\pi}{\rho(2M+\Delta)} \\ &= \sum_{n=1}^{\frac{\rho(2M+\Delta)\sqrt{V(2M+\Delta)}}{\pi}} \frac{1.1}{\rho(2M+\Delta)} = \frac{1.1}{\pi} \sqrt{V(2M+\Delta)} = 0.35 \sqrt{\frac{\Delta}{(2M+\Delta)^3}} \end{aligned} \quad (39)$$

Using $V_{scalar}(2M + \Delta) \approx V_{fermion}(2M + \Delta) = V(2M + \Delta)$, which is correct if $\Delta \ll M$. The total energy will be of the form:

$$E = \sqrt{M^2 + \frac{Const}{2M(2M + \Delta)^2}} + 0.35 \sqrt{\frac{\Delta}{(2M + \Delta)^3}} \quad (40)$$

The M^2 term is the rest energy of the planckon, the $\frac{Const}{2M(2M + \Delta)^2}$ term is the kinetic energy due to the uncertainty principle and the $0.35 \sqrt{\frac{\Delta}{(2M + \Delta)^3}}$ is the quantum self energy of the planckon.

Note that the contribution to the self energy comes mainly from trans-planckian modes i.e. modes with wavelength, which is lower than the Planck length.

3.4 Discussion On The Self Energy

The self energy expression (39) will now be examined. Expression (39) is not only finite but also small (since $\Delta \ll M$). This is of high importance since, otherwise, the black hole would not have tunnelled into the planckon due to energy conservation. Expression (39) was calculated for only 2 super-partner fields out of the total number of fields (this number should be about several hundreds and will denoted as C_f). For each super-multiplet the self energy expression should be proportional to (39) so the correct expression for the self energy has the form:

$$E_{self} = k \cdot C_f \sqrt{\frac{\Delta}{(2M + \Delta)^3}} \quad (41)$$

Where k is an unknown factor (which might also be negative) due to the contribution of each of the super-multiplets and C_f is the factor due to the total number of fields (or super-multiplets). The expression (41) is also small if one demands that:

$$\Delta < \frac{(2M)^3}{(k \cdot C_f)^2} \quad (42)$$

The last expression gives some limitations of the value of Δ , which was arbitrary up to now. The finiteness of the self energy is an important example for a scenario, in which, the infinite degeneracy of the planckon (which is proportional to the number of energy levels) does not give a divergent expression but a small finite expression because of the coupling (proportional to g_{00}). The same effect should happen in each order of quantum loop corrections in field theory hence rendering the effect of the infinite degeneracy of the planckon finite and no divergence will occur. It should be noted that super-symmetry had to be incorporated to achieve finite self-energy in the above example.

3.5 Physical Properties Of The Super-Symmetric Potential Well Model

As mentioned above this model has no horizon and no singularity (as expected from a quantum gravity theory).

The mass of the Planckon is approximately m_{pl} since the self energy is much smaller than the classical mass and the energy term that comes from the uncertainty effect.

There are approximately $g^{00} = e^{\frac{4M}{\Delta}}$ possible states, which are effectively degenerate, since the separation between the states is in the order of $\sqrt{g_{00}}$.

The only way for a field to interact with the internal degrees of freedom is to enter the volume inside the mass. The time for such an interaction to take place, for any observer who observes the interaction from outside the Planckon, is of the order of $\sim \frac{1}{\sqrt{g_{00}}} = e^{\frac{2M}{\Delta}}$, because of the gravitational time dilation due to the inner metric.

Effects, such as the time dilation, can make the production rate of a Planckon effectively zero (despite the Planckon's huge degeneracy), by rendering the Planckon coupling much smaller than the degeneracy. An example for such a scenario was presented by the self energy calculations in 3 and discussed in 3.4.

As a consequence of the Planckon vanishing coupling the only possibility to experimentally find evidence for a Planckon is through its gravitational effects since it has no charge (charges are emitted through tunnelling).

The main drawback of the spherical well model is the δ -function divergence of the energy-momentum tensor. The following model does not suffer from the same problem

but its self energy "diverges" as $\sqrt{g^{00}}$.

4 A Linear Model For The Planckon

In this model the total mass M is distributed linearly in the following way:

$$m(r) = \begin{cases} \frac{r-\Delta}{2} & 0 < r \leq 2M + \Delta \\ M & else \end{cases}$$

The energy momentum tensor behave as:

$$T_t^t(r) = \begin{cases} \frac{1}{8\pi r^2} & 0 < r \leq 2M + \Delta \\ 0 & else \end{cases}$$

The metric has the form:

$$-g^{rr} = \begin{cases} 1 & r \leq \Delta \\ \frac{\Delta}{r} & \Delta < r < 2M + \Delta \\ 1 - \frac{2M}{r} & 2M + \Delta \leq r \end{cases} \quad g_{00} = \begin{cases} e^{-\frac{4M}{\Delta}} & r \leq \Delta \\ \frac{\Delta}{r} e^{-\frac{2}{\Delta}(2M+\Delta-r)} & \Delta < r < 2M + \Delta \\ 1 - \frac{2M}{r} & 2M + \Delta \leq r \end{cases}$$

4.1 The Self Energy Of A Complex Scalar Field In The Linear Model

The effective potential a scalar field experiences due to the curved background is:

$$V_{scalar}(r) = \frac{1}{2r} \partial_r ((-g^{11}) g_{00}) = (\Delta r^{-3} - \Delta^2 r^{-4}) e^{-\frac{2}{\Delta}(2M+\Delta-r)} \quad \Delta \leq r \leq 2M + \Delta$$

To calculate the energy of the complex scalar field the WKB approximation has been used.

The Bohr-Sommerfeld quantization condition (up to some constant in the left hand side) is given by :

$$n\pi = \int_0^{\rho'} \sqrt{\omega^2 - V(\rho')} d\rho' = \frac{1}{\sqrt{\Delta}} \int_{\Delta}^r r' \sqrt{(r^{-3} - \Delta r^{-4}) e^{\frac{2}{\Delta}(r-r')} - (r'^{-3} - \Delta r'^{-4})} dr'$$

From the quantization condition one gets the density of states:

$$\frac{dn}{dr} = \frac{(2r^2 - 5\Delta r + 4\Delta^2)}{2\pi\Delta^{\frac{3}{2}}r^3} \int_{\Delta}^r \frac{r'^3 e^{\frac{2}{\Delta}(r-r')}}{\sqrt{(r-\Delta)r'^4 e^{\frac{2}{\Delta}(r-r')} - (r'-\Delta)r'^4}} dr'$$

The complex scalar self-energy is given by:

$$\begin{aligned} \langle E \rangle_{scalar} &= 2 \cdot \frac{1}{2} \sum \omega \approx \sum_{n=1}^{n_{\max}} \sqrt{V(r)} \simeq \int_1^{n_{\max}} \sqrt{V(r)} dn = \int_{\Delta}^{2M+\Delta} \sqrt{V(r)} \frac{dn}{dr} dr \\ &= \int_{\Delta}^{2M+\Delta} \frac{\sqrt{(r-\Delta)(2r^2-5\Delta r+4\Delta^2)} e^{-\frac{1}{\Delta}(2M+\Delta-r)}}{2\pi\Delta r^5} \left(\int_{\Delta}^r \frac{r'^3 e^{\frac{2}{\Delta}(r-r')}}{\sqrt{(r-\Delta)r'^4 e^{\frac{2}{\Delta}(r-r')} - (r'-\Delta)r'^4}} dr' \right) dr \end{aligned}$$

This expression diverges as $\sqrt{g^{00}} = e^{\frac{2M}{\Delta}}$ and again, in an attempt to obtain a finite expression, super-symmetry is invoked in the same way as in the spherical well model (adding the fermion super-partner of the complex scalar).

4.2 The Self Energy Of A Fermion In The Linear Model

The effective potential the fermion experiences, due to the curved background is:

$$V_{fermion} = W^2(\rho) = \frac{k^2}{r^2} g_{00} = \frac{\Delta}{r^3} e^{-\frac{2}{\Delta}(2M+\Delta-r)} \quad \Delta \leq r \leq 2M + \Delta$$

To calculate the energy of the fermion field the WKB approximation has been used in the same way as for the complex scalar.

From the bohr-sommerfeld quantization condition one gets the density of states:

$$\frac{dn}{dr} = \frac{2r - 3\Delta}{2\pi\Delta^{\frac{3}{2}}r^{\frac{5}{2}}} \int_{\Delta}^r \frac{r'^{\frac{3}{2}} e^{\frac{2}{\Delta}(r-r')}}{\sqrt{r'^3 e^{\frac{2}{\Delta}(r-r')} - r^3}} dr'$$

The total self energy of a fermion is:

$$\begin{aligned}\langle E \rangle &= 2 \cdot \frac{1}{2} \sum \omega \approx \sum_{n=1}^{n_{\max}} \sqrt{V(r)} \simeq \int_1^{n_{\max}} \sqrt{V(r)} dn = \int_{\Delta}^{2M+\Delta} \sqrt{V(r)} \frac{dn}{dr} dr \\ &= \int_{\Delta}^{2M+\Delta} \frac{(2r^2-3\Delta r)e^{-\frac{1}{\Delta}(2M+\Delta-r)}}{2\pi\Delta r^4} \left(\int_{\Delta}^r \frac{r'^{\frac{3}{2}} e^{\frac{2}{\Delta}(r-r')}}{\sqrt{r'^3 e^{\frac{2}{\Delta}(r-r')} - r^3}} dr' \right) dr\end{aligned}$$

This expression's divergent behavior is the same as for the complex scalar field.

4.3 The Total Self Energy Of The Super-Symmetric Linear Model

The total self energy for the super-symmetric linear model is given by:

$$\begin{aligned}\langle E \rangle &= \langle E \rangle_{boson} - \langle E \rangle_{fermion} \\ \xrightarrow{\Delta \ll r} & \int_{\Delta}^{2M+\Delta} \frac{((2r^2-5\Delta r)-(2r^2-3\Delta r))e^{-\frac{1}{\Delta}(2M+\Delta-r)}}{2\pi\Delta r^5} \left(\int_{\Delta}^r \frac{r'^{\frac{3}{2}} e^{\frac{2}{\Delta}(r-r')}}{\sqrt{r'^3 e^{\frac{2}{\Delta}(r-r')} - r^3}} dr' \right) dr \\ \sim & - \int_{\Delta}^{2M+\Delta} \frac{e^{-\frac{1}{\Delta}(2M+\Delta-r)}}{\pi r^4} \left(\int_{\Delta}^r \frac{r'^{\frac{3}{2}} e^{\frac{2}{\Delta}(r-r')}}{\sqrt{r'^3 e^{\frac{2}{\Delta}(r-r')} - r^3}} dr' \right) dr\end{aligned}$$

The energy is still divergent but the divergence is smaller by a factor of $\sim \frac{\Delta}{r}$, which is obviously not enough since the divergence is exponential.

The reason that the divergence is not totally eliminated is that the effective potential of the scalar and the fermion are the same only to the first order in $\frac{\Delta}{r}$.

4.4 Methods Of Reducing The Divergence

Several methods have been examined in order to reduce the divergence of the self-energy such as finding different geometries that will allow one to have some other parameters, besides Δ , to control the divergences. However as long as one keeps a linear section in the mass distribution, one ends up with similar divergences. Some attempts to reduce the divergence were to insert other consideration such as tunnelling³ and measurement theory considerations but they are not directly connected to the self energy and as such

³The divergence in the self energy expression may be eliminated by imposing a cutoff at $r=M$. The only reason to impose such a cutoff is due to tunnelling effect

can not help to solve the problem.

Another possibility for divergence reduction, which was not considered, is including in the self energy computation the whole gravity super-multiplet i.e. the gravitino related vacuum diagrams, which were not included in the computation.

4.5 Physical Properties Of The Super-Symmetric Linear Model

The linear model has all the physical properties of the spherical well model (see section 3.5). This model is also physical, since the metric is continuous and the energy-momentum tensor is not a delta-function, but a finite regular function.

Currently, the main drawback of the linear model is the self energy divergence. If not eliminated (or at least shown to be reduced by other vacuum diagrams that were not taken into account) this divergence will prevent the black hole from tunneling into the Planckon, due to energy conservation. The fact that the self energy of the spherical well model is finite, raises the hope, that a method can be found which will render the self-energy of the linear model finite (perhaps by considering contributions from the gravity super-multiplet as was mentioned in 4.4).

5 Discussion

It has been shown, that models exist, which have the general properties needed to make the planckon physically possible. These models render most of the arguments against the planckon non-relevant, especially the loop divergence arguments⁴.

The other argument mentioned in 1.3 is based on entropy bounds, which were derived using adiabatic processes such as lowering a box into a black hole. Arguments based on adiabatic continuous processes are irrelevant for the Planckon, as described here, for two main reasons:

⁴There is also the argument mentioned in 1.3 about the coupling of a soft graviton to a planckon anti-planckon, but this argument have no physical ground, since a soft planckon will not be able to create a planckon anti-planckon pair, because of energy conservation and the extrapolation of the interactions of gravitons from low-energy physics into planck scale energy physics is not valid, since there is possibly a new and different physics at the planck scale

- In general, entropy bounds only measure the difference of entropy of the systems caused by the process and not the initial entropy of the systems. The Planckon can hold a huge amount of information, while being involved in processes that change the total entropy of the whole system by a small amount.
- The specific models of the Planckon presented here and in [7, 8, 9] , where the process of a creation of a Planckon contains quantum processes such as tunnelling, cannot be described by continuous adiabatic processes such as the ones used for deriving the entropy bounds.

Another argument, which can be raised against the models described here, is that the Planckon should have infinite degeneracy (not just very high degeneracy). The argument goes as follows:

A black hole can swallow a Planckon. If a Planckon is the final state of a black hole it should keep the information of the black hole and the swallowed Planckon. The only way to achieve that goal is by requiring the Planckon to have infinite degeneracy.

A possible resolution can be obtained by considering black holes having an internal⁵ Planckon counter. The value of the counter is the number of swallowed Planckons. A black hole whose counter has the value N evaporates into $N+1$ Planckons when its energy reaches $N+1$ times the Planckon mass. This allows the Planckon to have very high yet finite degeneracy.

One can see that the models described in this paper solve the main problems of black hole remnants, although there are still many open questions:

1. **Making the spherical well model physical** - The main drawback of the spherical well model is the divergence of the energy-momentum tensor, since it has the form of a delta function (which in turn creates a discontinuity in the metric). Possible extensions to the given model may include extensions adjustments that will make the metric continuous by allowing the energy-momentum tensor to be distributed over a finite non-vanishing region, while keeping the self energy from diverging (most likely by small perturbations of the energy-momentum tensor)
2. **Making the self energy of the linear model finite** - The main drawback

⁵The meaning of internal is that it does not affect the metric generated by the black hole so that black hole theory will not have to be changed

of the linear model is the divergence of its self energy. Elimination of the self energy might be achieved by methods described in 4.4.

3. **Determining the value of Δ and M** - The ratio $\frac{\Delta}{M}$ is of great importance for the given models, but nowhere in the models are the exact values of neither Δ nor M calculated. To calculate the value of M , one needs a dynamical model of the Hawking radiation near the planck scale. The value of Δ poses more problems, since the models do not give any method of determining its size. Also, it has no apparent scale and its size may be much lower than the planck scale, raising the question of the minimal length scale in physics. The most probable way to determine Δ is by finding the minimum value of the energy, which as for now is not within reach, due to the number of different fields involved up to the planck scale.
4. **Finding a model with minimal self energy** - Since two models were introduced, one of which diverges while the other gives a finite small result, a variational principle might be used to claim, that a model with minimal self energy exists. Finding such a model is closely connected to the problem of determining the value of Δ and M , since their values and ratio determine the self energy.

If the Planckon exists it should dominate the planck scale spectrum. As such the models may provide hints, as to what properties planck scale fields are expected to have and might help to shed some light on some of the unsolved problems in quantum gravity and astrophysics such as the information paradox, dark matter, cosmological constant and different questions related to planck scale physics [7, 8, 9].

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